On gauge couplings and thresholds in Type I Gepner models and otherwise

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ABSTRACT: We derive general formulae for tree level gauge couplings and their one-loop thresholds in Type I models based on genuinely interacting internal $\mathcal{N} = 2$ SCFT's, such as Gepner models. We illustrate our procedure in the simple yet non-trivial instance of the Quintic. We briefly address the phenomenologically more relevant issue of determining the Weinberg angle in this class of models. Finally we initiate the study of the correspondence between 'magnetized' or 'coisotropic' D-branes in Gepner models and twisted representations of the underlying $\mathcal{N} = 2$ SCA.

Contents

1. Introduction

Type I strings and their close relatives have received a great deal of attention in the past few years (see *e.g.* [1]-[7] for comprehensive reviews).

Although their systematization was already achieved in the early 90's [8]-[16], including the possibilities of minimally coupling R-R p-form potentials and reducing the rank of the Chan-Paton group by turning on a quantized NS-NS antisymmetric tensor background [15, 16], the geometric description in terms of D-branes and Ω-planes [20, 21], pioneered in [22, 23] has definitely consecrated this framework as the most promising one to embed Particle Physics in String Theory. Simple instances of chiral model based on toroidal orbifolds [24]- [29] with or without intersecting branes [30]-[33], that are T-dual to magnetized branes [34]- [38], represent a useful guidance for more sophisticated and hopefully realistic constructions

that may require *inter alia* (non) commuting open string Wilson lines or their closed string dual constructions [16]-[19].

The important issues of supersymmetry breaking [39]-[43] and moduli stabilization [44]-[56] have been tackled with some success. Interactions at tree (disk and sphere) level [57]-[65] have been studied in some detail. One-loop thresholds for the gauge couplings have been computed [66]-[71] and some steps beyond one-loop have been made [72]. More recently it has been argued that large extra dimensions naturally emerge in this approach [73]-[75]. In these cases, predictions for processes with missing energy at near future colliders [76]-[79] have been put forward.

Following the by now standard construction of RCFT's on surfaces with crosscaps and boundaries [80]-[82], open and unoriented models based on genuinely interacting internal $\mathcal{N} = 2$ SCFT's, such as Gepner models [83]-[85], have been constructed in [86]-[94] and accurately scanned in order to test the possibility of accommodating the Standard Model [92, 93]. Indeed, contrary to perturbative heterotic strings, it is rather contrived if not impossible to embed interesting Grand Unified Theories (GUT's) in perturbative Type I strings. Exceptional groups, such as $E(6)$ are ruled out by a theorem of Marcus and Sagnotti's [95, 96], and the same applies to spinorial representation of Orthogonal groups, such as $SO(10)$. One could then look for chiral GUT's based on unitary groups such as $SU(5)$. Although, with some effort, one can find reasonable $U(5)$ three generation models with Higgses in the adjoint and in the $5 + \bar{5}$, these models turn out to be unrealistic since only the Yukawa couplings $\phi_{\mathbf{5}}\psi_{\mathbf{5}}\chi_{\overline{\mathbf{10}}}$ are allowed by $U(1)$ charge conservation. The Yukawa couplings $\phi_{\bar{\mathbf{5}}}\psi_{\overline{\mathbf{10}}}\chi_{\overline{\mathbf{10}}}$, though $SU(5)$ invariant, are forbidden by $U(1)$ charge conservation and by the impossibility of generating the necessary antisymmetric tensor ϵ_{ijklm} as Chan-Paton factor, *i.e.* taking traces of matrices [97, 98]. Barring non-perturbative effects that can significantly change this state of affairs but whose study is only in its infancy, the best one can achieve is some L-R symmetric extension of the SM or a Pati-Salam generalization thereof, together with some (anomalous) $U(1)$'s. The role of the latter has been carefully studied recently [99, 100] and we will not add much here.

Aim of the present paper is to derive general formulae for the (non abelian) gauge couplings and their one-loop thresholds in Type I models based on type II Gepner models. Quite remarkably we will find elegant and compact formulae valid whenever the internal CFT enjoys $\mathcal{N} = 2$ worldsheet SCI. This allows to construct a parent type II (B) theory which is supersymmetric and corresponds to the compactification on a CY 3-fold (or $K3$ or T^2). Depending on the brane and Ω plane configuration, the resulting type I model may enjoy spacetime susy. Indeed, as it was first advocated in [101] and it was exploited more recently in the context of Black-Hole physics [102] and intersecting D-brane models [103], it is possible that each pair of branes enjoys some susy (common to the Ω-planes that can in fact coincide with some of the stacks) which is not the same for all pairs. Even in this case, one-loop amplitudes would look supersymmetric and some of the threshold corrections could be reliably computed by means of our formulae.

After illustrating our formulae in the case of a Type I model on the Quintic with gauge group $SO(12) \times SO(20)$, we address the possibility of determining the Weinberg angle in phenomenologically more promising models in this class. This is tightly related to the

embedding of the $U(1)_Y$ hypercharge generator in the Chan-Paton group [93].

Finally, we briefly discuss the issue of computing some four-point amplitudes along the lines of [50] and initiate the program of studying and classifying 'magnetized' or 'coisotropic' D-branes in Gepner models. As it was argued in [51], these correspond to twisted representations of the underlying $\mathcal{N} = 2$ superconformal algebra (SCA). We will not explicitly consider the interesting possibility of constructing models with large extra dimensions based on (freely acting) orbifolds of $K3 \times T^2$ at Gepner points for $K3$ [52]. Neither we will consider turning on closed string fluxes (metric torsion, NS-NS 3-form flux and R-R fluxes) and their effect of non-trivial warping of the geometry [104]. Being optimistic, this would at least require resorting to alternative approaches [105], where supersymmetry properties are manifest such as the pure spinor formalism [106], or the hybrid formalism [107] or other manifestly supersymmetric formalisms [72, 108].

We leave to future work a more thorough analysis of gauge couplings and thresholds in phenomenologically viable models as well as the study of other important ingredients in the low-energy effective action.

2. $\mathcal{N} = 2$ SCFT and Gepner models

We start with a general discussion of the worldsheet properties of supersymmetric vacuum configurations for (open and unoriented) strings.

2.1 $\mathcal{N} = 2$ SCFT

As it was shown by Banks and Dixon [109] in order to have spacetime susy in $D = 4$, the underlying SCFT must enjoy at least $\mathcal{N} = 2$ superconformal invariance on the worldsheet. In addition to the stress tensor T and the two spin $3/2$ supercurrents G^+ and G^-, the $\mathcal{N} = 2$ superconformal algebra includes also a $U(1)$ R-symmetry current J. *A priori* the $\mathcal{N} = 2$ worldsheet supercurrents can acquire arbitrary phases under parallel transport around non-trivial cycles *i.e.*

$$G^{\pm}(e^{2\pi i}z) = e^{2\pi i \nu_{\pm}} G^{\pm}(z) \tag{2.1}$$

where we can choose $|\nu_{\pm}| \leq 1/2$. As a consequence, their modes are labelled by $r_{\pm} \in \mathbf{Z} + 1/2 + \nu_{\pm}$. If $\nu_+ + \nu_- \neq 0$ the current J has non-integer modes and one finds what is called a 'twisted' representation of the $\mathcal{N} = 2$ SCA. In the rest of the paper we shall consider only the case $\nu_{\pm} = \pm \nu$. As a consequence, the current J has integer modes and the two supercurrents $G^{\pm}$ have $U(1)$ charge ± 1 respectively. Different values of ν are isomorphic and they are connected by the 'spectral flow' induced by the action of the unitary operator

$$\mathcal{U}_{\nu} = \exp(2\pi i \nu J_0) \,. \tag{2.2}$$

The cases $\nu = 0$ and $\nu = \pm 1/2$ correspond to the NS and R sector, respectively, and they are related by one unit of spectral flow *i.e.* by $\mathcal{U}_{\pm 1/2}$. These are singled out as the only boundary conditions compatible with the 'reality' of the $\mathcal{N} = 1$ supercurrent

$$G_{\mathcal{N}=1} = G^+ + G^- \,, \tag{2.3}$$

that couples to the worldsheet gravitino. Bosonizing the $U(1)$ current as

$$J = \mathrm{i}\sqrt{\frac{c}{3}}\partial H \;, \tag{2.4}$$

the spectral flow is related to the spacetime supercharges [110] that in $D = 4$ read[1]

$$\mathcal{Q}_\alpha = \int \frac{dz}{2\pi\mathrm{i}}e^{-\varphi/2}S_\alpha e^{\frac{\mathrm{i}}{2}\sqrt{\frac{c}{3}}H} \quad , \quad \mathcal{Q}_{\dot\alpha} = \int \frac{dz}{2\pi\mathrm{i}}e^{-\varphi/2}C_{\dot\alpha}e^{-\frac{\mathrm{i}}{2}\sqrt{\frac{c}{3}}H} \tag{2.5}$$

where φ is the superghost boson and $S_\alpha, C_{\dot\alpha}$ are spin fields of opposite chirality. Depending on the spin of the state, locality of the OPE of $\mathcal{Q}_\alpha, \mathcal{Q}_{\dot\alpha}$ with the vertex operators determines the correct quantization condition for the $U(1)$ R-charge.

In $D = 4$, *i.e.* for $c_{int} = 9$, the vertex operator for a vector boson is

$$V_{-1} = a_\mu(p)\psi^\mu e^{-\varphi}e^{\mathrm{i}q_v\sqrt{\frac{3}{c}}H}e^{\mathrm{i}pX} \tag{2.6}$$

locality requires $q_v = 0\,(\mathrm{mod}\,2)$. The vertex operator for a scalar is

$$V_{-1} = \phi(p)e^{-\varphi}\widehat{\Psi}_{q_o}e^{\mathrm{i}q_o\sqrt{\frac{3}{c}}H}e^{\mathrm{i}pX} \;, \tag{2.7}$$

where $\Psi_{q_o} = \widehat{\Psi}_{q_o}e^{\mathrm{i}q_o\sqrt{\frac{3}{c}}H}$ is a primary field in the NS sector with $U(1)$ charge $q = q_o$ and dimension $h = (1 + p^2)/2$. Locality requires $q_o = 1\,(\mathrm{mod}\,2)$. Massless scalars correspond to (anti)chiral primaries with $h = 1/2$ so that $q_o = \pm 1$, *a priori* $0 \le q_{CPO} < c/3$. For the (massless) LH spinor, the vertex operator is

$$V_{-1/2} = u^\alpha(p)S_\alpha e^{-\varphi/2}\widehat{\Sigma}_{q_s}e^{\mathrm{i}q_s\sqrt{\frac{3}{c}}H}e^{\mathrm{i}pX}, \tag{2.8}$$

where $\Sigma_{q_s} = \widehat{\Sigma}_{q_s}e^{\mathrm{i}q_s\sqrt{\frac{c}{3}}H}$ is a primary field in the R sector with $U(1)$ charge $q = q_s$ and dimension $h = (c/24) + (p^2/2)$ and locality requires $q_s = +3/2\,(\mathrm{mod}\,2)$. Massless LH spinors correspond to R groundstates (RGS) with $h = c/24 = 3/8$ and $q_s = +3/2$, so that $h_{\widehat{\Sigma}} = (c/24) - (3q_s^2/2c)$, *a priori* $-c/6 \le q_{RGS} \le c/6$. For the (massless) RH spinor, the vertex operator is

$$V_{-1/2} = v_{\dot\alpha}(p)e^{-\varphi/2}C^{\dot\alpha}\widehat{\Sigma}^\dagger_{q_c}e^{\mathrm{i}q_c\sqrt{\frac{3}{c}}H}e^{\mathrm{i}pX} \tag{2.9}$$

where locality requires $q_c = -3/2\,(\mathrm{mod}\,2)$.

2.2 Unitary $\mathcal{N} = 2$ minimal models

Unitary $\mathcal{N} = 2$ minimal models are known to form a discrete series [110]. They are equivalent to the quotients $SU(2)_k \times U(1)_2/U(1)_{k+2}$, so that the central charge is given by

$$c(k) = \frac{3k}{k+2} \;, \tag{2.10}$$

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[1]We mostly focus on the case $D = 4$ corresponding to $c_{int} = 9$.

where k is a positive integer. The $\mathcal{N} = 2$ primary fields $\Phi_{m,s}^{\ell(k)}$ are labelled by three quantum numbers $0 \leq \ell \leq k$, $-(k+1) \leq m \leq k+2$ and $s = 0, \pm 1, 2$, with $\ell + m + s = 0 \,(\mathrm{mod}\,2)$. By the field identifications

$$\Phi_{m,s}^{\ell(k)} = \Phi_{m,s+4}^{\ell(k)} = \Phi_{m+2(k+2),s}^{\ell(k)} = \Phi_{m+k+2,s+2}^{k-\ell(k)} \tag{2.11}$$

one can restrict the values of (ℓ, m, s) to the 'standard' range $s = 0, \pm 1$, $\ell \leq [k/2]$, $-(k+1) < m \leq k + 2$.

The spectrum of conformal dimensions and $U(1)$ charges are given by[2]

$$h(\ell, m, s) = \frac{\ell(\ell+2) - m^2}{4(k+2)} + \frac{s^2}{8} \quad (\mathrm{mod}\,1) , \tag{2.12}$$

$$q(m, s) = \frac{m}{k+2} - \frac{s}{2} \quad (\mathrm{mod}\,2) . \tag{2.13}$$

Every $\mathcal{N} = 2$ minimal model can be decomposed into a parafermionic theory and a free $U(1)$ boson, so that

$$T = T_{PF} - \frac{1}{2}\partial H \partial H \quad , \quad G^{\pm} = \sqrt{\frac{2c(k)}{k}} \psi_{PF}^{\pm} e^{\pm i\sqrt{\frac{3}{c(k)}}H} \quad , \quad J = i\sqrt{\frac{c(k)}{3}}\partial H \tag{2.14}$$

one has

$$\Phi_{m,s}^{\ell(k)} = \hat{\Phi}_{m-s}^{\ell(k)} e^{i\gamma_{m,s}^k H} \tag{2.15}$$

where

$$\gamma_{m,s}^k = \sqrt{\frac{k+2}{k}} \left(\frac{m}{k+2} - \frac{s}{2} \right) = \sqrt{\frac{3}{c(k)}} q(m,s) . \tag{2.16}$$

Unitarity requires $h_{\hat{\Phi}} \geq 0$ i.e. $h_{\Phi} \geq 3q^2/2c(k)$ ('unitary parabola'). Moreover, in the NS sector $(s = 0, 2)$

$$h_{NS} \geq \frac{1}{2}|q_{NS}| . \tag{2.17}$$

The inequality is saturated by (anti) chiral primary operators (CPO) corresponding to $m = \pm\ell$ and $s = 0$ with $|q_{CPO}| \leq c(k)/3$ that satisfy

$$G_{-1/2}^{+}|h = q/2;\ q\rangle_{CPO} = 0 \quad \text{or} \quad G_{-1/2}^{-}|h = -q/2;\ q\rangle_{CPO\dagger} = 0 . \tag{2.18}$$

In the R sector $(s = \pm 1)$

$$h_R \geq \frac{c(k)}{24} . \tag{2.19}$$

The inequality is saturated by Ramond ground-states (RGS) corresponding to $m = \pm(\ell+1)$ and $s = \pm 1$ with $|q_{RGS}| \leq c(k)/6$ that satisfy

$$G_0^{\pm}|h = c(k)/24;\ q\rangle_{RGS} = 0 \tag{2.20}$$

and contribute to the Witten index $\mathcal{I}_W = Tr(-)^F$.

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[2]In the rest of this paper we shall always tacitly assume the (mod 1) and (mod 2) conditions for h and q.

2.3 Gepner models

Gepner models [83, 85] are tensor products of r minimal $\mathcal{N} = 2$ models quotiented by a subgroup of the discrete symmetries that keeps only the states with quantized $U(1)$ charge and sectors in which the $\mathcal{N} = 1$ worldsheet supercurrent

$$G_{\mathcal{N}=1} = G^+ + G^- = \sum_{i=1}^{r}(G_i^+ + G_i^-) \tag{2.21}$$

is well defined *i.e.* transforms covariantly, acquiring at most a sign under parallel transport around non-trivial cycles. The latter condition looks at first as a merely worldsheet requirement, dictated by consistency of the coupling of $G_{\mathcal{N}=1}$ to the worldsheet gravitino, but actually it is a necessary condition for BRS invariance and decoupling of negative norm states. The $U(1)$ charge quantization is equivalent to the condition for spacetime supersymmetry, whose chiral action ('spectral flow') is only well defined on states with quantized $U(1)$ charges. Indeed bosonizing the $U(1)$ current one finds

$$\mathcal{J} = i\sqrt{\frac{c}{3}}\partial H = i\sum_i \sqrt{\frac{c_i}{3}}\partial H_i \ , \tag{2.22}$$

so that

$$H = i\sum_i \sqrt{\frac{c_i}{c}}\partial H_i \ . \tag{2.23}$$

For our latter purposes it is crucial to further investigate the decomposition of the individual terms in the $\mathcal{N} = 1$ worldsheet supercurrent (2.21)

$$G_i^\pm = \widehat{G}_i^\pm e^{\pm i\sqrt{\frac{3}{c_i}}H_i} \tag{2.24}$$

where $\widehat{G}_i^\pm = \psi_{PF,i}^\pm$ are NS primary fields of dimension

$$h_{\widehat{G}_i^\pm} = \frac{3}{2} - \frac{3}{2c_i} = 1 - \frac{1}{k_i} \tag{2.25}$$

that can be identified with the fundamental $\mathbf{Z}_{k_i}$ parafermions defining the coset $SU(2)_{k_i}/U(1)$. In particular for $k = 1$ one has $\widehat{G}^\pm = \mathbf{1}$, while for $k = 2$ one finds $\widehat{G}^+ = \widehat{G}^- = \psi$, the 'real' fermion of the Ising model. The first 'non-trivial' case is $k = 3$ (relevant for the quintic) where $\widehat{G}^+ = \rho$ and $\widehat{G}^- = \rho^\dagger$ with $h_\rho = h_{\rho^\dagger} = 2/3$.

Moreover, as stated above, one should demand $\nu_i = \nu_j = \nu_{st}$ for any i and j, with st standing for $G_{st} = \psi_\mu \partial X^\mu$, in order for $G_{tot} = G_{st} + G_{int}$ to be well defined. This is at the heart of the so-called β_i $\mathbf{Z}_2^r$-projections which were first introduced by Gepner [83] in analogy with what was done in free fermionic models. Experience with orbifolds and magnetized and/or intersecting D-branes suggest that 'twisted' representations that have been thrown out of the door may snick in through the window. Indeed, one can preserve covariance of the $\mathcal{N} = 1$ worldsheet supercurrent $G_{tot} = G_{st} + G_{int}$ by changing the boundary conditions of the internal bosonic coordinates X_{int}^I and at the same time by implementing the same ('contragradient') change of the internal fermionic coordinates

Ψ_{int}^I. Inspection of (2.24) suggests that a shift of the boson H_i can be 'compensated' by a twist of the parafermion $\psi_{PF,i}$. We will further elaborate on this observation in Section 6. For the time being let us focus on standard 'untwisted' UIR's.

For tensor product theories, primary fields can be written as $\Phi_{h,q} = \prod_i \Phi_{h_i,q_i}$ with

$$h = \sum_{i=1}^r h_i = \sum_{i=1}^r \left[\frac{\ell_i(\ell_i+2) - m_i^2}{4(k_i+2)} + \frac{s_i^2}{8} \right] \tag{2.26}$$

and

$$q = \sum_{i=1}^r q_i = \sum_{i=1}^r \left[\frac{m_i}{k_i+2} - \frac{s_i}{2} \right] . \tag{2.27}$$

In order to restrict the spectrum to the states on which $G_{\mathcal{N}=1}$ acts consistently, as a whole, one has to combine states that impose the same boundary condition on each term in (2.24). The resulting $\mathbf{Z}_2^r$ projection can be achieved in different ways. We follow the orbit procedure developed by Eguchi, Ooguri, Taormina and Yang [85].

In our conventions, the total susy charge (which is proportional to the spectral flow operator) reads

$$Q = S_2 \prod_i \Phi_{-1,-1}^{(i),0} . \tag{2.28}$$

It has total charge $c/6 = \sum_i c_i/6 = 3/2$ in $D = 4$ i.e. for $c = 9$ and, barring the spin field S_2 (with helicity $\lambda = +1/2$ and scaling dimension $1/8$), dimension $c/24 = \sum_i c_i/24 = 3/8$. The supercurrent in each subtheory reads

$$G_i = \Phi_{0,2}^{(i),0} . \tag{2.29}$$

Then, given the Highest Weight State (HWS) $\mathcal{X}_V^{HWS}$ in the V_2 part of a 'resolved' susy character (e.g. for the identity sector $\mathcal{X}_V^{HWS} = \prod_i \mathcal{X}_{0,0}^{(i),0}$) satisfying

$$q_V^{HWS} = 0 \ (\text{mod } 2) \tag{2.30}$$

the action of $Q^n \prod_i G_i^{p_i}$ maps it into a state with

$$q(n, p_i) = q_V^{HWS} + n\frac{c}{6} + \sum_i p_i = q_V^{HWS} + n\frac{3}{2} + \sum_i |p_i| , \tag{2.31}$$

since $p_i = |p_i| \ (\text{mod } 2)$ for $p_i = 0, 1$. Setting

$$K = \text{l.c.m.}\{4, 2(k_i+2)\} \tag{2.32}$$

we can write for the complete Gepner model characters (orbits)[3]

$$\chi_I = \sum_{n=0}^{K-1} (-)^n Q^n \prod_{i=1}^r \sum_{p_i=0,1} (O_2 G_i)^{p_i} (V_2 \mathcal{X}_{V,I}^{HWS}) . \tag{2.33}$$

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[3]When all the levels are odd, one has in addition to divide (2.33) by 2.

For some purposes, it is convenient to manifestly separate the contribution of the non compact space-time super-coordinates and write *e.g.* for $D = 4$

$$\chi_I = V_2 \mathcal{X}_I^V + O_2 \mathcal{X}_I^O - S_2 \mathcal{X}_I^S - C_2 \mathcal{X}_I^C , \qquad (2.34)$$

where V_2, O_2, S_2, C_2 are $SO(2)$ characters at level one and the minus signs take into account spin and statistics. Supersymmetry entails $\chi_I = 0$ for all I. Since by assumption (see eq. (2.30))

$$q_V^{HWS} = \sum_i \left[\frac{m_i}{k_i + 2} - \frac{s_i}{2} \right] = 0 \ (\text{mod} \, 2) \qquad (2.35)$$

for the Ramond sector internal characters one finds

$$\mathcal{X}_I^S = \sum_{m=0}^{\frac{K}{4}-1} \left[\sum_{\substack{p_i=0,1 \\ \sum_i p_i = 0 (mod\,2)}} \prod_i \chi_{m_i-4m-1,s_i-1+2p_i}^{\ell_i(k_i)} + \sum_{\substack{p_i=0,1 \\ \sum_i p_i = 1 (mod\,2)}} \prod_i \chi_{m_i-4m+1,s_i+1+2p_i}^{\ell_i(k_i)} \right] , \qquad (2.36)$$

and

$$\mathcal{X}_I^C = \sum_{m=0}^{\frac{K}{4}-1} \left[\sum_{\substack{p_i=0,1 \\ \sum_i p_i = 0 (mod\,2)}} \prod_i \chi_{m_i-4m+1,s_i+1+2p_i}^{\ell_i(k_i)} + \sum_{\substack{p_i=0,1 \\ \sum_i p_i = 1 (mod\,2)}} \prod_i \chi_{m_i-4m-1,s_i-1+2p_i}^{\ell_i(k_i)} \right] . \qquad (2.37)$$

In principle the resulting 'supersymmetric' characters (2.34) have 'length' (total number of terms) $L = 2^r K$. However, although neither Q nor G_i independently have fixed points, *i.e.* they act freely, it may happen that some orbits are shorter due to field identifications. It can be shown, that this can happen only when some of the k_i are even and that the short orbits are always twice shorter $L_{short} = 2^{r-1} K$, so as far as only the spectrum of conformal dimensions is considered, one has simply to halve the expressions above. The situation becomes more involved if modular transformations are considered. In this case one has to resolve the fixed point ambiguity which amounts to 'split' the representation encoded in the supersymmetric character into two independent representations, possibly conjugate to one another, that have to be labelled by an additional quantum number.

2.4 Open descendants

In these cases, the parent 'oriented' closed string theory is based on a perturbative spectrum encoded in the one-loop torus partition function

$$\mathcal{T} = \sum_{I,J} \mathcal{T}_{IJ} \chi^I \bar{\chi}^J \qquad (2.38)$$

where $q = \exp(2\pi i \tau)$ and the characters χ_I provide a fully resolved unitary representation of the modular group. The non-negative integers $\mathcal{T}_{IJ}$ are tightly constrained by modular invariance. Denoting by $I = 0$ the character of the identity representation of the RCFT, $\mathcal{T}_{00} = 1$ implies the presence of only one graviton in the massless spectrum. Simple solutions

are: the charge conjugation modular invariant $\mathcal{T}_{IJ} = \mathcal{C}_{IJ}$ ('Cardy'), and the 'diagonal' modular invariant $\mathcal{T}_{IJ} = \delta_{IJ}$.

The massless spectrum is encoded in those combinations for which $h_I = \bar{h}_J = 1/2$. Since V_2 already corresponds to $h_V = 1/2$, the only massless contribution of this kind comes from $\mathcal{X}_0^V$ which corresponds to the identity of the internal CFT. Other massless bosons come from O_2 combined with $h_I^{int} = \bar{h}_J^{int} = 1/2$. In Gepner models these are in one to one correspondence with chiral (c) and anti-chiral (a) primary operators with $q_I = \pm \bar{q}_J = \pm 1$, respectively. In type IIB, (c,c) states and their conjugate (a,a) states give rise to $h_{2,1}$ $\mathcal{N} = 2$ vector multiplets, comprising two NS-NS scalars and one R-R vector, while (c,a) states and their conjugate (a,c) states give rise to $h_{1,1} + 1$ $\mathcal{N} = 2$ hyper-multiplets, comprising two NS-NS scalars and two R-R 'axions' (dual to two-forms). The special Kähler 'geometry' of the vector multiplets is tree level exact since corrections in $g_s = \langle \phi \rangle$ are forbidden. Indeed the dilaton ϕ is part of the universal hypermultiplet and as such it cannot have neutral couplings to vector multiplets. The same argument applies to worldsheet instanton corrections that depend on the sizes of the holomorphic cycles governed also by scalars in hypermultiplets. On the contrary, the dual quaternionic geometry of the hypermultiplets can be corrected both perturbatively and non-perturbatively.

The generalized Ω-projection is encoded in the Klein bottle amplitude

$$\mathcal{K} = \sum_I K_I \chi^I(q\bar{q}) \tag{2.39}$$

where $K_I = \mathcal{T}_{II}$ (mod 2) determines in particular which massless fields are retained. Typically (but not necessarily) both vector multiplets and hypers produce 'chiral' (or rather linear) multiplets. Yet if one splits $h_{2,1}$ into $h_{2,1}^+ + h_{2,1}^-$ where the apex indicates an extra possible sign, constrained by the so-called crosscap constraint and associated to some internal anticonformal involution, one can show that the resulting unoriented spectrum contains $h_{2,1}^+$ chiral multiplets and $h_{2,1}^-$ abelian vector multiplets, comprising R-R vectors, in addition to $h_{1,1}^+ + h_{1,1}^-$ chiral/linear multiplets.

The open string partition function is given by

$$\mathcal{A} = \sum_{I,a,b} A_{Ia\bar{b}} n^a \bar{n}^b \chi^I \ , \tag{2.40}$$

where n^a is the number of 'generalized' D-branes of type a and A_{ab}^I are integer multiplicities constrained by the quadratic equations

$$\sum_{b\bar{b}} A_{Ia\bar{b}} \delta^{\bar{b}b} A_{Jb\bar{c}} = \sum_K N_{IJ}^K A_{Ka\bar{c}} \ , \tag{2.41}$$

where N_{IJ}^K are the fusion rule coefficients, which can be expressed in terms of the fully resolved S_{IJ} via Verlinde formula. Finally, the Möbius strip Ω-projection reads

$$\mathcal{M} = \sum_{I,a,b} M_{Ia} n^a \hat{\chi}^I \ , \tag{2.42}$$

where $M_{Ia} = A_{Iaa}$ (mod 2) and $\hat{\chi}^I$ denote a real basis of characters introduced in [12]. We remind that the arguments of the Annulus and Möbius amplitudes are different, namely

$\tau_A = it/2$, $\tau_M = \tau_A + 1/2$. In what follows (unless essential), we shall systematically omit the τ dependence in the characters.

Worldsheet covariance conditions between the direct channel, exposing the projection of the closed string spectrum ($\mathcal{K}$) or the open string spectrum ($\mathcal{A}$ and $\mathcal{M}$), and the transverse channel (which is exposing the closed string exchange between boundaries and crosscaps) puts tight constraints on the coefficients K_I, $A_{Ia\bar{b}}$ and M_{Ia}.

For the case of the charge conjugation modular invariant $\mathcal{T}_{IJ} = \mathcal{C}_{IJ}$ one has as many boundaries (i.e. n's) as characters, and one solution (known as Cardy's solution) is given by

$$A_{IJK} = N_{IJK} \quad , \quad K_I = Y_{I00} \quad , \quad M_{IJ} = Y_{JI0} \, . \tag{2.43}$$

Here N_{IJK} are the fusion rule coefficients, while Y_{IJK} are (possibly negative) integers given by

$$Y_{IJK} = \sum_L \frac{S_{IL} P_{JL} P_{KL}}{S_{0L}} \, , \tag{2.44}$$

where $P = T^{1/2} S T^2 S T^{1/2}$ is the Möbius strip modular matrix implementing the transformation $(\mathrm{i}t + 1)/2 \rightarrow (\mathrm{i} + t)/2t$. The respective boundary and crosscap reflection coefficients are

$$\mathrm{B}_I = \frac{\sum_J S_{IJ} n^J}{\sqrt{S_{0I}}} \quad , \quad \Gamma_I = \frac{P_{0I}}{\sqrt{S_{0I}}} \, . \tag{2.45}$$

3. Tree level gauge couplings

As suggested in [111], the 'generalized' Born-Infeld action for branes in (non)geometric backgrounds that admit a (rational) CFT description can be extracted from factorization of the one-loop annulus amplitude in the transverse channel. This applies to Gepner models which are expected to correspond to special (often non singular) points in the moduli space of CY compactifications, where the Kähler and the complex structure moduli take string scale VEV's i.e. $R \approx \sqrt{\alpha'}$ and the supergravity approximation might be questionable. Yet, the worldsheet string description is fully reliable in perturbation theory. In fact nonperturbative effects in R^2/α' and even in $1/g_s$ may be systematically incorporated.

3.1 Tadpole cancellation and gauge couplings

A consistent space time interpretation, requires the absence of tadpoles for massless states, which schematically reads

$$\mathrm{B}_I + 2^{D/2} \Gamma_I = 0 \, , \qquad \forall I \, : \, h_I = 1/2 \quad . \tag{3.1}$$

Although NS-NS tadpoles only signal an instability of the chosen configuration, it has proved very hard to dispose of them by vacuum redefinition [112, 113]. On the other hand, R-R tadpoles are associated to anomalies [111]. In fact R-R tadpole conditions are more restrictive than simply chiral anomaly cancellation that is associated to R-R tadpoles in sectors with non-vanishing Witten index [111]. Actually some left-over anomalies involving $U(1)$ factors in the Chan-Paton group can be disposed of by the combined effect of axions,

playing the role of Stückelberg fields, and generalized Chern-Simons couplings [99]. We will henceforth assume that a solution to the R-R tadpole conditions has been found, *i.e.* a consistent choice of n_a has been made. Supersymmetry would then imply the absence of NS-NS tadpoles.[4]

Tree level dependence of gauge couplings on massless closed string moduli can be determined by considering a three-point amplitude on the disk with one closed string insertion in the bulk and two massless open string insertions (vector bosons) on the boundary. The boundary is mapped to the brane a. The amplitude reads

$$\langle c V_A^{(0)}(x_1) \int V_A^{(0)}(x_2) c\bar{c} V_{ReZ}^{(-1,-1)}(z, \bar{z}) \rangle \tag{3.2}$$

for the CP even coupling and

$$\langle c V_A^{(0)}(x_1) \int V_A^{(-1)}(x_2) c\bar{c} V_{ImZ}^{(-1/2,-1/2)}(z, \bar{z}) \rangle \tag{3.3}$$

for the CP odd coupling. For the open string insertions one can use the gauge boson vertex operators introduced previously. For the closed string insertion one has to combine scalar vertex operators for the Left and Right movers. Using $SL(2)$ invariance one can put $z = \mathrm{i}$, $\bar{z} = -\mathrm{i}$ and $x_1 = \infty$. Integration over x_2 produces a constant and the overall factor is exactly $B_a^I Tr_a(T_1 T_2)$ where the Chan Paton factor has replaced n_a that appears for an empty boundary. This measures

$$\mathrm{B}_a^I = \left.\frac{\partial f_a}{\partial Z^I}\right|_{Z_I=0} \tag{3.4}$$

where we have assumed that the rational (Gepner) point corresponds to $Z_I = 0$ and

$$f_a = \frac{\mathrm{i}\vartheta_a}{2\pi} + \frac{4\pi}{g_a^2} , \tag{3.5}$$

is the gauge kinetic function for branes of type a.

One arrives at the above conclusion by 'factorization' of the one-loop non-planar amplitude in the transverse channel. If χ_I is a massless character which starts with the complex scalar field Z_I, one can conclude that the tree-level gauge coupling is given by

$$f_a(Z_I) = f_a(Z_I = 0) + \mathrm{B}_a^I Z_I , \tag{3.6}$$

to lowest order in Z_I. In particular the dilaton dependence, measuring the tension of the brane, is given by $n^a B_a^0 Z_0$ where $Z_0 = S$ to adhere to standard notation. In fact if Z_I contains a pseudoscalar axion, shifting under some (gauged) PQ symmetry, this is the full story, *i.e.* f is at most linear in Z_I. This is always true in sectors with non-vanishing Witten index. The dependence on Z_I belonging to sectors with vanishing Witten index can be more involved and they can appear in the one-loop threshold corrections. Moreover, multiplicities in sectors with $\mathcal{N} = 1$ susy *i.e.* non vanishing Witten index are excluded by

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[4]As mentioned in the introduction, it is sufficient that each pair of branes preserves some supersymmetry in order for this to be true.

our assumption that fixed point ambiguities have been resolved. On the contrary, sectors with $\mathcal{N} = 2$ susy entail a twofold degeneracy at least. Scalars from sectors with $\mathcal{N} = 4$ susy can contribute to the tree level gauge couplings but not to the one-loop thresholds. Anyway, it is remarkable how a low-energy coupling can directly probe the structure of the underlying RCFT coded in the B_a^I, that in turn depend on the choice of K_I, A_{Iab} and M_{Ia} and the 'resolved' matrices S_{IJ} and P_{IJ}. In particular the value of the Weinberg angle at the string scale is related to the ratio of the real parts of the gauge kinetic function for $SU(2)_W$ and the properly normalized $U(1)_Y$,

$$\tan^2 \vartheta_W = \frac{g_Y^2}{g_W^2} = \frac{\mathcal{R} f_W}{\mathcal{R} f_Y} = \frac{\mathcal{R} \mathrm{B}_W^I Z_I}{\mathcal{R} \mathrm{B}_Y^J Z_J} \tag{3.7}$$

where as above Z's runs over all closed string moduli fields and, obviously, in order for the formula to be predictive at all, one has to assume the closed string moduli have been stabilized by some flux or non-perturbative effect.

4. One-loop thresholds corrections

The purpose of this Section is to obtain explicit and (relatively) simple formulae for the one-loop threshold corrections to the gauge couplings. In four dimensions, gauge couplings run logarithmically as a result of massless particles in the loops. Massive states, such as generalized KK modes or genuine string excitations, induce threshold corrections Δ_a in the form of

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_a^2(M)} + \frac{b_a}{8\pi^2} \log\left(\frac{\mu}{M}\right) + \Delta_a \tag{4.1}$$

where b_a is the coefficient of the one-loop β function. Threshold corrections signal the dependence on the light scalar fields in the macroscopic theory of the mass scale M at which the matching with the microscopic theory is performed.

We will follow the strategy pioneered in [66, 67] and successfully applied to type I orbifolds in [68, 101], to generic type I vacuum configurations in [101] and to intersecting brane models in [69], based on the background field method. We give only a very brief summary of the arguments. For details see *e.g.* [71]. The method consists in applying a small abelian constant magnetic field in some spacetime directions, computing the effect of such an integrable deformation and then extracting the quadratic term in the one-loop effective action.

Following [71], we turn on an abelian magnetic field in spacetime directions 2 and 3, leaving unmodified the light cone directions 0 and 1,

$$F_{\mu\nu} = \delta_{[\mu}^2 \delta_{\nu]}^3 f \mathcal{H} \tag{4.2}$$

where $\mathcal{H}$ is one of the generators of the unbroken CP group. Depending on the embedding of $\mathcal{H}$ in the CP group one finds different behaviors. To avoid complications we will focus only on the case in which $\mathcal{H}$ is a generator of a non-abelian and thus non-anomalous factor labelled by a. Expanding the Annulus and the Möbius amplitudes $\mathcal{A}_a(f)$ and $\mathcal{M}_a(f)$ to

second order in f, one finds schematically for the one-loop gauge threshold for the group to which belongs $\mathcal{H}$ [67, 68]:

$$\Delta_a = \int \frac{dt}{t} (\mathcal{A}_a''(0) + \mathcal{M}_a''(0)) = \int \frac{dt}{4t} \mathcal{B}_a(t) \,, \tag{4.3}$$

where the prime denotes the derivative with respect to f. The expression is IR divergent, signalling the running of the gauge couplings, and needs regularization that, for non abelian gauge groups simply amounts to replacing $\mathcal{B}_a(t)$ with $\mathcal{B}_a(t) - b_a$ with b_a the on-loop β function coefficient.

The presence of the magnetic field implies that the space-time characters entering the Annulus and Möbius amplitudes will have a non-zero z argument.

$$\chi_I(z,\tau) = V_2(z,\tau)\mathcal{X}_I^V(0,\tau) + O_2(z,\tau)\mathcal{X}_I^O(0,\tau) - S_2(z,\tau)\mathcal{X}_I^S(0,\tau) - C_2(z,\tau)\mathcal{X}_I^C(0,\tau) \,, \tag{4.4}$$

where I labels the different orbits / sectors in the theory. Let us stress that $z \neq 0$ ($f \neq 0$) in eq. (4.4) breaks supersymmetry, so the characters $\chi_I(z,\tau)$ are not identically zero anymore. The second derivative with respect to f in eq. (4.3) translates into a second derivative with respect to z of the characters $\chi_I(z,\tau)$. Since only the space time is z dependent, one finds

$$\mathcal{B}_I(0) = V_2''(0)\mathcal{X}_I^V(0) + O_2''(0)\mathcal{X}_I^O(0) - S_2''(0)\mathcal{X}_I^S(0) - C_2''(0)\mathcal{X}_I^C(0) \,, \tag{4.5}$$

where here, and in the rest of this Section, prime denotes a derivative in z. Putting all pieces together

$$\mathcal{B}_a(t) = \sum_{I,b} A_{ab}^I n^b \mathcal{B}_I(t) + \sum_I M_a^I \hat{\mathcal{B}}_I(\hat{t}) \,, \tag{4.6}$$

where A_{ab}^I, M_a^I are integer multiplicities and n^b are the number of branes in each stack.

4.1 Thresholds from $\mathcal{N} = 2$ SCFT

Due to the very complicated form of the internal characters $\mathcal{X}_I^\lambda$ in Gepner models, the above expression for $\mathcal{B}_I(0)$ is not very useful. In order to rewrite it in a more tractable form, let us introduce the supersymmetric $SO(2) \times U(1)_R$ spacetime characters $v, \phi, \phi^\dagger$ defined by [114, 25]

$$v(z,y) = V_2(z)\xi_0(y) + O_2(z)\xi_3(y) - S_2(z)\xi_{+3/2}(y) - C_2(z)\xi_{-3/2}(y) \,,$$
$$\phi(z,y) = V_2(z)\xi_{-2}(y) + O_2(z)\xi_{+1}(y) - S_2(z)\xi_{-1/2}(y) - C_2(z)\xi_{+5/2}(y) \,, \tag{4.7}$$
$$\phi^c(z,y) = V_2(z)\xi_{+2}(y) + O_2(z)\xi_{-1}(y) - S_2(z)\xi_{-5/2}(y) - C_2(z)\xi_{+1/2}(y) \,.$$

Here $\xi_p(y)$ which encode the coupling to the total R-symmetry charge $J_R = \sum_i J_R^{(i)}$, are given by

$$\xi_p(y) = \frac{1}{\eta} \sum_n q^{\frac{1}{6}(p+6n)^2} e^{2\pi i y(p+6n)} \tag{4.8}$$

and satisfy

$$12\pi i \, \partial_\tau \left(\xi_p(y)\eta \right) = \eta \, \partial_y^2 \xi_p(y) \,. \tag{4.9}$$

In any SUSY compactification to $D = 4$, the characters can be decomposed according to

$$\chi_I(z, y, \tau) = v(z, y, \tau)\hat{\chi}_I^v(\tau) + \phi(z, y, \tau)\hat{\chi}_I^\phi(\tau) + \phi^c(z, y, \tau)\hat{\chi}_I^{\phi_c}(\tau), \qquad (4.10)$$

where $\hat{\chi}^\Lambda$ are characters of $(\mathcal{N} = 2)/U(1)_R$. It is quite remarkable and crucial for our subsequent analysis that

$$v(z, y = z/3) = 0, \qquad \phi(z, y = z/3) = 0, \qquad \phi^c(z, y = z/3) = 0, \qquad (4.11)$$

for any z thanks to theta functions identities (*cf. e.g.* [115]). Then it follows immediately that also

$$\chi_I(z, z/3, \tau) = 0, \qquad (4.12)$$

for all values of I, z and τ. This tantalizingly suggest the possibility of building more general supersymmetric 'magnetized' aka 'coisotropic' branes. We will come back to this issue in a later section.

Taking the first derivative with respect to τ and the second derivative with respect to z of eqs. (4.7) for $y = z/3$ and using that $4\pi i \partial_\tau \chi_\lambda^{SO(2)}(z) = \partial_z^2 \chi_\lambda^{SO(2)}(z)$ (up to an irrelevant η) as well as eq. (4.9) one finds

$$\mathcal{B} + \frac{1}{3}\mathcal{A} = 0, \qquad \mathcal{B} + \frac{1}{9}\mathcal{A} = -\frac{2}{3}\mathcal{C}. \qquad (4.13)$$

Here $\mathcal{B}$ collectively denotes terms with second derivative of $\chi_\lambda^{SO(2)}(z)$ (*i.e.* which contribute to the thresholds), $\mathcal{A}$ denotes terms with second derivative of $\xi_p(z/3)$ and $\mathcal{C}$ terms with two first derivatives. Eliminating $\mathcal{A}$, one finds $\mathcal{B} = -\mathcal{C}$ and after substituting in eqs. (4.10,4.7) one then gets

$$\begin{aligned}\mathcal{B}_I(z, z/3) = -[&V_2'(z)(\mathcal{X}_I^V)'(z/3) + O_2'(z)(\mathcal{X}_I^O)'(z/3) \\ &- S_2'(z)(\mathcal{X}_I^S)'(z/3) - C_2'(z)(\mathcal{X}_I^C)'(z/3)],\end{aligned} \qquad (4.14)$$

where $\mathcal{X}_I^\lambda(z/3)$ denotes the character valued internal partition function in the relevant sector of the orbit I.

For $z = 0$, $V_2'(0) = O_2'(0) = 0$, while $S_2'(0) = -C_2'(0) = i\theta_1'/2 = i\pi\eta^3$ that cancels a similar factor in the denominator. So finally we obtain

$$\mathcal{B}_I = \frac{d}{dy}(\mathcal{X}_I^S(y) - \mathcal{X}_I^C(y))\bigg|_{y=0}. \qquad (4.15)$$

We stress that this general formula is valid for any susy compactification to $D = 4$. In the particular case of Gepner models, it can be additionally simplified. Indeed, using eqs. (2.36) and (2.37), we can write

$$\mathcal{B}_I = \frac{d}{dy}\mathcal{W}_I(y)\bigg|_{y=0}, \qquad (4.16)$$

where $\mathcal{W}_I(y)$ in the sector I is given by

$$\mathcal{W}_I(y) = (-1)^{r + \sum_i \frac{m_i}{k_i+2}} \sum_{n=0}^{K/2-1} (-1)^{n(r-1)} \prod_{i=1}^r \mathcal{W}_{m_i-2n-1}^{\ell_i}(y), \qquad (4.17)$$

where

$$\mathcal{W}_{m_i-2n-1}^{\ell_i}(y) = \chi_{m_i-2n-1,1}^{\ell_i}(y) - \chi_{m_i-2n-1,-1}^{\ell_i}(y) = Tr_{\mathcal{H}_{i,n}}[(-)^F e^{2\pi i y J_o} q^{L_o-c_i/24}] \quad (4.18)$$

is called elliptic index. For $y = 0$ it is a constant, and since only the Ramond groundstates contribute

$$\mathcal{W}_{m_i-2n-1}^{\ell_i}(y=0) = \mathcal{I}_{m_i-2n-1}^{\ell_i} = \delta_{m_i-2n-1,\ell_i+1} - \delta_{m_i-2n-1,-\ell_i-1}, \quad (4.19)$$

where both deltas are computed mod $2(k_i + 2)$. Thus the derivative in eq. (4.16) reduces to

$$\mathcal{W}_I'(y=0) = (-1)^{r+\sum_i \frac{m_i}{k_i+2}} \sum_{n=0}^{K/2-1} (-1)^{n(r-1)} \sum_{j=1}^{r} (\mathcal{W}_{m_j-2n-1}^{\ell_j})'(y=0) \prod_{\substack{i=1 \\ i \neq j}}^{r} \mathcal{I}_{m_i-2n-1}^{\ell_i}. \quad (4.20)$$

This expression can be further simplified with the help of eq. (4.19).

Starting from the expression for $\chi_{l,m}^{NS^+}$, given in [52], one can derive the expression for $\mathcal{W}_m^\ell = \chi_{l,m}^{R^-}$ by a shift of the argument $z \to z + (\tau+1)/2$

$$\mathcal{W}_m^\ell(z) = \chi_{m,1}^\ell(z) - \chi_{m,-1}^\ell(z) \quad (4.21)$$

$$= \frac{e^{i\pi\left(\frac{\ell+m+1}{k+2}-1\right)} \theta_1(z,\tau) \, \theta\begin{bmatrix} -\frac{\ell+1}{k+2}+\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(0,(k+2)\tau) \, \eta^3((k+2)\tau)}{\eta^3(\tau) \, \theta\begin{bmatrix} \frac{\ell+m+1-(k+2)}{2(k+2)} \\ \frac{1}{2} \end{bmatrix}(z,(k+2)\tau) \, \theta\begin{bmatrix} \frac{-\ell+m-1+(k+2)}{2(k+2)} \\ \frac{1}{2} \end{bmatrix}(z,(k+2)\tau)}$$

$$= \frac{e^{-i\pi z\left(\frac{m}{k+2}\right)} q^{\frac{(\ell+1)^2-m^2}{4(k+2)}} \eta^3\left((k+2)\tau\right) \theta_1(z,\tau) \, \theta_1\left((\ell+1)\tau,(k+2)\tau\right)}{\eta^3(\tau) \, \theta_1\left(z-\frac{\ell-m+1}{2}\tau,(k+2)\tau\right) \theta_1\left(z+\frac{\ell+m+1}{2}\tau,(k+2)\tau\right)}.$$

It is immediate that $\mathcal{W}_m^\ell(0) = 0$ unless $m = \ell + 1$ or $m = -(\ell+1)$, $\mathcal{W}_{\ell+1}^\ell(0) = 1$ and $\mathcal{W}_{-\ell-1}^\ell(0) = -1$. Moreover one can show that

$$\sum_{\ell=0}^{k} \mathcal{W}_{\ell+1}^\ell(z) = \frac{\theta_1(\frac{k+1}{k+2}z,\tau)}{\theta_1(\frac{1}{k+2}z,\tau)}. \quad (4.22)$$

Let us denote

$$a = \frac{\ell+1}{(k+2)}, \quad b = \frac{\ell+1-m}{2(k+2)}, \quad c = \frac{\ell+1+m}{2(k+2)}. \quad (4.23)$$

Then for the derivatives $(\mathcal{W}_m^\ell)'(0)$ one finds

$$(\mathcal{W}_{\ell+1}^\ell)'(0) = (\mathcal{W}_{-\ell-1}^\ell)'(0) = \frac{d}{dy} \ln\left(\theta\begin{bmatrix} \frac{1}{2}-a \\ \frac{1}{2} \end{bmatrix}(y,(k+2)\tau)\right)\bigg|_{y=0}, \quad (4.24)$$

while if $m \neq \ell+1, -\ell-1$

$$(\mathcal{W}_m^\ell)'(0) = 2\pi i \, q^{\frac{(\ell+1)^2-m^2}{4(k+2)}} \frac{\mathcal{P}_k(0)^2 \, \mathcal{P}_k(a) \, \mathcal{P}_k(1-a)}{\mathcal{P}_k(b) \, \mathcal{P}_k(1-b) \, \mathcal{P}_k(c) \, \mathcal{P}_k(1-c)} \,, \tag{4.25}$$

where

$$\mathcal{P}_k(\alpha) = \prod_{n=1}^\infty (1 - q^{(k+2)(n-\alpha)}) \,. \tag{4.26}$$

4.2 Thresholds in toroidal orbifolds

For completeness and for comparison, let us summarize here known formulae for the thresholds corrections to gauge couplings in Type I (magnetized) toroidal orbifolds. It is known that some Gepner models, $e.g.$ $(k=1)^9$ or $(k=2)^6$ models in $D=4$, correspond to toroidal orbifolds at special points in their moduli spaces. Formulae in this section would then apply to these cases. For brevity we only discuss the contribution of $\mathcal{N}=1$ supersymmetric sectors. Expanding the annulus and Möbius strip amplitudes to quadratic order in the background field f and summing over spin structures by means of

$$\sum_{\alpha\beta} c_{\alpha\beta} \frac{\theta''[{}^\alpha_\beta](0)}{\eta^3} \prod_I \frac{\theta[{}^\alpha_\beta](u^I)}{\theta_1(u^I)} = 2\pi \sum_I \frac{\theta_1'(u^I)}{\theta_1(u^I)} \,, \tag{4.27}$$

give

$$\mathcal{B}_a^{\mathcal{N}=1}(t) = \frac{i}{\pi} \sum_b \mathcal{I}_{ab} N_b \sum_I \frac{\theta_1'(u_{ab}^I|\tau_A)}{\theta_1(u_{ab}^I|\tau_A)}$$

$$\hat{\mathcal{B}}_a^{\mathcal{N}=1}(t) = -\frac{i}{\pi} \sum_a \mathcal{I}_{a\tilde{a}} N_a \sum_I \frac{\theta_1'(u_{a\tilde{a}}^I|\tau_M)}{\theta_1(u_{a\tilde{a}}^I|\tau_M)} \,, \tag{4.28}$$

where

$$u_{ab}^I = \kappa v_{ab}^I + \epsilon_{ab}^I \tau \,, \tag{4.29}$$

satisfy $\sum_I u_{ab}^I = 0$ and take into account both the orbifold projection κv_{ab}^I ($e.g.$ $\kappa = 1, ..., n$ for $\Gamma = \mathbf{Z}_n$) and the mass shift ϵ_{ab}^I due to magnetic flux or intersections at angle. The one-loop β-function coefficients can be extracted from the IR limit of (4.28).

In order to perform the integral and compute Δ_a in magnetized tori ($v_{ab}^I = 0$), it is convenient to switch to the transverse channel, where one finds

$$\Delta_a^{\mathcal{N}=1} = \frac{1}{2\pi} \sum_{a,b} \mathcal{I}_{ab} N_b \sum_I \int_0^\infty \frac{\theta_1'(\epsilon_{ab}^I|i\ell)}{\theta_1(\epsilon_{ab}^I|i\ell)} \, d\ell \,,$$

$$\hat{\Delta}_a^{\mathcal{N}=1} = -\frac{1}{2\pi} 2\mathcal{I}_{a\tilde{a}} \sum_I \int_0^\infty \frac{\theta_1'(\epsilon_{0a}^I|i\ell+1/2)}{\theta_1(\epsilon_{0a}^I|i\ell+1/2)} \, d\ell \,. \tag{4.30}$$

Series expansion

$$\frac{\theta_1'(\epsilon|\tau)}{\theta_1(\epsilon|\tau)} \pi \cot(\pi\epsilon) + 2 \sum_{k=1}^\infty \zeta(2k)\epsilon^k(E_{2k}(\tau) - 1) \quad, \tag{4.31}$$

where $\zeta(2k) = (2\pi)^{2k}|B_{2k}|/(2k)!$ and $E_{2k}(\tau)$ is an Eisenstein series with modular weight $2k$, expose potentially divergent terms that eventually cancel thanks to (NS-NS) tadpole cancellation, for the non-anomalous $\mathcal{H}$, with $Tr(\mathcal{H}) = 0$. The finite terms boil down to integrals of the form (see for example [69])

$$\int_0^\infty d\ell \sum_{k=1}^\infty 2\zeta(2k)\epsilon^k(E_{2k}(i\ell) - 1) = -\pi \log\left[\frac{\Gamma(1-\epsilon)}{\Gamma(1+\epsilon)}\right] + 2\pi\epsilon\gamma_E , \qquad (4.32)$$

$$\int_0^\infty d\ell \sum_k 2\zeta(2k)\epsilon^k(E_{2k}(i\ell + 1/2) - 1) = -\pi \log\left[\frac{\Gamma(1-2\epsilon)}{\Gamma(1+2\epsilon)}\right] + 2\pi\epsilon\gamma_E . \qquad (4.33)$$

Actually the last contributions, linear in ϵ, drop after summing over the three internal directions in supersymmetric cases.

Summing the various contributions one finally gets

$$\Delta_a^{\mathcal{N}=1} = -\sum_b \mathcal{I}_{ab} N_b \sum_I \log\left[\frac{\Gamma(1-\epsilon_{ab}^I)}{\Gamma(1+\epsilon_{ab}^I)}\right] \quad ,$$

$$\hat{\Delta}_a^{\mathcal{N}=1} = \sum_a 2\mathcal{I}_{a\tilde{a}} \sum_I \log\left[\frac{\Gamma(1-\epsilon_{aa}^I)}{\Gamma(1+\epsilon_{aa}^I)}\right] \quad , \qquad (4.34)$$

where $\epsilon_{aa}^I = 2\epsilon_{ao}^I$.

Field dependent thresholds corrections from $\mathcal{N} = 2$ sectors with vanishing Witten index ($u_{ab}^I = 0$ for some $I = \parallel$, so that $u_{ab}^{\perp,1} = -u_{ab}^{\perp,2}$) are much easier to compute since they correspond to BPS saturated couplings. We refrain from doing so explicitly here. $\mathcal{N} = 4$ sectors ($u_{ab}^I = 0$ for all I) do not contribute threshold corrections to the gauge couplings.

5. Examples

Once the general formula has been derived, in order to compute explicit thresholds one has to put together various bits and pieces.

First one has to fix the integer multiplicities in the annulus and Moebius amplitudes compatibly with tadpole cancellation.

Second one has to choose a non-abelian group and identify the sectors of the open string spectrum which are charged. We neglect possibly anomalous $U(1)$'s since the above formulae do not immediately apply. In fact they rather compute the masses of the gauge bosons via their mixings with R-R axions.

Third one has to perform the integral over t. This was done above for magnetized tori and it is possible for some contributions (from fully massless sectors) in type I Gepner models as well.

Let us discuss what happens in various dimensions.

5.1 Models in $D = 8$

In $D = 8$ supersymmetric models correspond to compactifications on 2-tori. The two derivative effective action is tree level exact because of susy. Some four derivative terms

such as F^4 are 1/2 BPS saturated. Starting from the seminal paper by Bachas and Fabre [67], one-loop threshold corrections to these and other BPS saturated have been used as tests of various string dualities. For a comprehensive review see [116].

5.2 Models in $D = 6$

Threshold corrections in $D = 6$ are topological in the sense that only massless states can contribute. Indeed in theories with $\mathcal{N} = (1,0)$ susy the gauge couplings can only depend on the VEV's of scalar that belong to tensor multiplets and not to hypermultiplets because of susy. In perturbative heterotic models the only tensor multiplet contains the dilaton and this produces the standard dependence of the gauge coupling from the string coupling. All the remaining moduli, either charged or neutral, belong to hypemultiplets. In type I constructions [12] various neutral tensor multiplets are present whose scalar components belong to the NS-NS sector. In principle gauge couplings may depend on them. There is a tight connection with anomaly related couplings as required by the generalized mechanism of anomaly cancellation.

After compactification to $D = 4$ on a 2-torus one gets $\mathcal{N} = 2$ theories whose gauge kinetic function is 1/2 BPS saturated. Only generalized KK modes contribute to the threshold. Generalized compactifications á la Scherk-Schwarz with freely acting orbifolds preserving $\mathcal{N} = 1$ may lead to interesting applications of our analysis in connection with large extra dimensions.

5.3 Models in $D = 4$. The Quintic : $(k = 3)^5$ model

The simplest non trivial case is a Type I model on the Quintic [87, 88, 52]. It is based on the diagonal modular invariant that puts fewer tadpole constraints than the charge conjugation modular invariant. Indeed, in the transverse channel only two massless sectors can propagate. The identity and the sector containing the unique (c,a) massless state (unique deformation of the Kähler structure). To cancel tadpoles one can introduce so-called B-type branes and in particular one can build a model with $SO(12) \times SO(20)$ Chan-Paton group. Though non chiral, the model serves as a non trivial illustration of our procedure.

The annulus partition function is given by

$$\mathcal{A} = \frac{1}{2}(n_0^2 + n_1^2)\chi_A + (\frac{1}{2}n_1^2 + n_0 n_1)\chi_B \ , \tag{5.1}$$

where $n_0 = 12$ and $n_1 = 20$. The Möbius strip projection reads

$$\mathcal{M} = -\frac{1}{2}(n_0 + n_1)\hat{\chi}_A + \frac{1}{2}n_1\hat{\chi}_B \ . \tag{5.2}$$

Here χ_A and χ_B are given by

$$\chi_A = \frac{1}{5}\left[(\chi_I)^5\right]^{\text{susy}} \ ,$$
$$\chi_B = \frac{1}{5}\left[(\chi_I)^4 \chi_{II}\right]^{\text{susy}} \ , \tag{5.3}$$

where χ_I and χ_{II} are defined as

$$\chi_I = \frac{1}{2}\left(\chi_{0,0}^0 + \chi_{0,2}^0 + \chi_{2,0}^0 + \chi_{2,2}^0 + \chi_{4,0}^0 + \chi_{4,2}^0 + \chi_{6,0}^0 + \chi_{6,2}^0 + \chi_{8,0}^0 + \chi_{8,2}^0\right), \quad (5.4)$$

$$\chi_{II} = \frac{1}{2}\left(\chi_{1,0}^1 + \chi_{1,2}^1 + \chi_{3,0}^1 + \chi_{3,2}^1 + \chi_{5,0}^1 + \chi_{5,2}^1 + \chi_{7,0}^1 + \chi_{7,2}^1 + \chi_{9,0}^1 + \chi_{9,2}^1\right), \quad (5.5)$$

in terms of the $\mathcal{N} = 2$, $k = 3$ characters $\chi_{m,s}^\ell$.

The massless spectrum is given by $\mathcal{N} = 1$ vector multiplets in $Adj[SO(20) \times SO(12)] = (\mathbf{190} + \mathbf{66})$ plus four chiral multiplets in the $(\mathbf{20}, \mathbf{12})$ and as many in the $(\mathbf{210}, \mathbf{1})$. One can thus easily compute the β functions for $SO(20)$ and $SO(12)$ and get

$$\beta_{SO(20)} = 3(20 - 2) - 4(12 + (20 + 2)) = -82$$
$$\beta_{SO(12)} = 3(12 - 2) - 4(20) = -50 \quad (5.6)$$

both gauge couplings are IR free.

From eqs. (4.16-4.22) we find:

$$\mathcal{B}_A = -5(\mathcal{W}_1^0)' - 5(\mathcal{W}_9^0)' - 30(\mathcal{W}_5^0)' + 20(\mathcal{W}_3^0)' + 20(\mathcal{W}_7^0)', \quad (5.7)$$

and

$$\mathcal{B}_B = 8(\mathcal{W}_1^0)' + 8(\mathcal{W}_9^0)' - 4(\mathcal{W}_2^1)' - 4(\mathcal{W}_8^1)' +$$
$$8(\mathcal{W}_5^0)' - 12(\mathcal{W}_3^0)' - 12(\mathcal{W}_7^0)' + 6(\mathcal{W}_0^1)' + (\mathcal{W}_4^1)' + (\mathcal{W}_6^1)'. \quad (5.8)$$

These derivatives can be computed with the help of eqs. (4.23-4.26). In particular for the contributions relevant for the β functions are

$$(\mathcal{W}_1^0)' = (\mathcal{W}_9^0)' = \frac{3}{5}\mathrm{i}\pi + \dots \quad (5.9)$$

$$(\mathcal{W}_2^1)' = (\mathcal{W}_8^1)' = \frac{1}{5}\mathrm{i}\pi + \dots \quad (5.10)$$

Contributions of fully massless sectors can be computed by means of (4.24) and integrated by means of (4.33). The contributions to the thresholds that involve one massive subsector can be computed by means of (4.25). We have not yet been able to find a simple way to integrate the result as for the fully massless sectors.

6. Magnetized aka coisotropic D-branes

In toroidal or orbifold compactifications one can easily impose 'generalized' boundary conditions that correspond to turning on a constant magnetic field on the worldvolume of the D-brane

$$[\partial X^i - R_{aj}^i \bar{\partial} X^j]|a\rangle_F = 0 \quad, \quad [\psi^i - \mathrm{i}\eta R_{aj}^i \bar{\psi}^j]|a\rangle_F = 0 \quad (6.1)$$

where $\eta = \pm 1$, depending on the sector, and the orthogonal matrix (in the frame basis) reads

$$R_{aj}^i = [\delta_{ak}^i - F_{ak}^i][\delta_{aj}^k + F_{aj}^k]^{-1} \quad (6.2)$$

$$- 19 -$$

R_{aj}^i can be diagonalized in a complex a-dependent basis Z^I, Z_I^*, so that

$$\partial Z^I = e^{2\pi i \nu_a^I} \bar{\partial} Z^I \tag{6.3}$$

as a result the modes of Z^I are shifted according to $n^I \to n^I + \nu_a^I$. A similar analysis applies to the complex fermions Ψ^I, Ψ_I^* such that $G = \partial Z_I^* \Psi^I + \partial Z^I \Psi_I^*$ (a-independent!!). When several stacks of magnetized branes are present, the rotation matrices R_a and R_b for different stacks would not commute in general. When $[R_a, R_b] = 0$ for all a and b, all the magnetic fields are *parallel*, otherwise $[R_a, R_b] \neq 0$ and the magnetic fields are *oblique*. Performing appropriate T-dualities on magnetized D9-branes one ends up with intersecting magnetized D-branes aka coisotropic D-branes. For parallel fields appropriate T-dualities lead to intersecting D-branes with no magnetization aka isotropic branes.

We would like to extend this analysis to compactifications based on genuinely interacting $\mathcal{N} = 2$ SCFT.

For simplicity one can consider Gepner models first. In this case the worldsheet supercurrent is given by

$$G = \sum_i [\psi_i^{PF} e^{i\sqrt{\frac{3}{c_i}}H_i} + \psi_i^{PF,\dagger} e^{-i\sqrt{\frac{3}{c_i}}H_i}] \quad . \tag{6.4}$$

There are two classes of boundary conditions preserving the diagonal $\mathcal{N} = 2$ SCA commonly called of A and B type. A-type boundary conditions imply

$$[\psi_i^{PF} - i\eta \bar{\psi}_i^{PF,\dagger}]|b\rangle_A = 0 , \quad [e^{i\sqrt{\frac{3}{c_i}}H_i} - e^{-i\sqrt{\frac{3}{c_i}}\bar{H}_i}]|b\rangle_A = 0 \tag{6.5}$$

and correspond to D-branes wrapping middle homology cycles (*i.e.* Special Lagrangian submanifolds) or generalized bound-states thereof.

B-type boundary conditions imply

$$[\psi_i^{PF} - i\eta \bar{\psi}_i^{PF}]|b\rangle_B = 0 , \quad [e^{i\sqrt{\frac{3}{c_i}}H_i} - e^{i\sqrt{\frac{3}{c_i}}\bar{H}_i}]|b\rangle_B = 0 \tag{6.6}$$

and correspond to D-branes wrapping even-dimensional homology cycles (*i.e.* complex submanifolds) or generalizations thereof.

One can envisage the possibility of imposing symmetry breaking boundary conditions such as

$$[\psi_i^{PF} - i\eta e^{2\pi i \nu_b^i} \bar{\psi}_i^{PF,\dagger}]|b\rangle_{\tilde{A}} = 0 , \quad [e^{i\sqrt{\frac{3}{c_i}}H_i} - e^{-2\pi i \nu_b^i} e^{-i\sqrt{\frac{3}{c_i}}\bar{H}_i}]|b\rangle_{\tilde{A}} = 0 \tag{6.7}$$

or

$$[\psi_i^{PF} - i\eta e^{2\pi i \nu_b^i} \bar{\psi}_i^{PF}]|b\rangle_{\tilde{B}} = 0 , \quad [e^{i\sqrt{\frac{3}{c_i}}H_i} - e^{-2\pi i \nu_b^i} e^{i\sqrt{\frac{3}{c_i}}\bar{H}_i}]|b\rangle_{\tilde{B}} = 0 \tag{6.8}$$

that should naturally correspond to D-branes wrapping submanifolds with non trivial magnetic fluxes and thus would deserve the name of 'coisotropic' D-branes in this context. More pragmatically the boundary conditions combine a shift in the $U(1)$ charge lattice with a compensating 'rotation' of the complex parafermions so as to preserve the diagonal $\mathcal{N} = 2$ SCA. In cases where several factors are isomorphic (*i.e.* have the same k) additional

'permutations' are possible in the boundary conditions leading to what have been called 'permutation' branes. The open string excitations of this more general class of D-branes belong to twisted representations of $\mathcal{N} = 2$ SCA that are known to exist for any real values of ν_b^i. Spacetime supersymmetry imposes further constraints [117, 115]. A detailed study of this class of branes is deferred to future work. Suffice it to say that including this new class of branes enormously widens the possibilities of accomodating interesting chiral models in Type I Gepner models.

For the time being let us check the validity of the above interpretation for the phenomenologically uninteresting case of $D = 8$, *i.e.* to $\mathbf{T}^2$ compactifications [118, 119], where a precise dictionary exist between the standard bosonic and fermionic coordinates X, ψ and parafermions ψ^{PF} and free boson H. Indeed for the $(1, 1, 1)$ model $c = 1 + 1 + 1 = 3$ and $H = \sum_i H_i / \sqrt{3}$ and

$$\Psi = e^{iH} , \quad \partial Z = \frac{1}{\sqrt{3}} \sum_i e^{i(H - \sqrt{3}H_i)} \tag{6.9}$$

while for the $(2, 2, 0)$ model $H = \sum_i H_i / \sqrt{2}$

$$\Psi = e^{iH} , \quad \partial Z = \frac{1}{\sqrt{2}} \sum_i \psi_i e^{i(H - 2H_i)} . \tag{6.10}$$

Finally, for the $(4, 1, 0)$ model $H = (\sqrt{2}H_1 + H_2)/\sqrt{3}$

$$\Psi = e^{iH} , \quad \partial Z = \frac{1}{\sqrt{2}} \left[\hat{\Psi}_{3/4} e^{i(H - \sqrt{\frac{2}{3}}H_1)} + e^{i(H - \sqrt{3}H_2)} \right] . \tag{6.11}$$

Switching on a non vanishing $\nu_b^i \neq 0$ is tantamount to turning on a magnetic field or, equivalently after T-duality, rotating the brane wrt the fundamental cell of the $\mathbf{T}^2$.

7. Concluding remarks

We have derived very compact and elegant formulae that allow one to determine the tree level gauge couplings and the one-loop thresholds in Type I or similar compactifications based on genuinely interacting $\mathcal{N} = 2$ SCA, such as Gepner models but not only. We have then given some explicit example for the non-abelian factors in the Chan-Paton gauge group. In view of [99, 100] the analysis of anomalous $U(1)$ factors may reserve for us new interesting possibilities. Moreover the computation of four vector boson scattering amplitudes at one-loop seems at reach, since the threshold encode the structure called $\mathcal{E}$. The other irreducible structure $\mathcal{F}$ require some more work.The analysis might be significantly simplified resorting to the hybrid formalism proposed by Berkovits.[5]

We have then briefly discussed how to generalize the standard boundary conditions so as to describe magnetized aka coisotropic D-branes. This new class of branes may open new paths not only to the construction of viable Type I models but also to the generation of non-perturbative effects, *i.e.* D-brane instantons, mediated by magnetized or coisotropic

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[5]M.B. would like to thank Nathan Berkovits for clarifying discussions on this issue.

ED-branes. It is in fact more than natural to expect that ED-branes wrapping the same cycle as a given stack of branes, including magnetization, are equivalent to standard gauge instantons for the resulting effective theory, while all other ED-branes generate stringy non perturbative phenomena.

Clearly before even contemplating stringy instanton effects in these backgrounds one should reliably compute tree level Yukawas and Kähler potential for the open string excitations. We hope to report on these issues soon although the perspectives of making reasonable predictions for the Cabibbo angle in this context are much weaker than for the Weinberg angle. It would also be interesting to study models with large extra dimensions á la Aldazabal et al [120, 121]or even non-susy models with supersymmetric partition functions. As mentioned in the introduction the final goal would be to stabilize all moduli and break susy in a controllable way. This may not forgo understanding better, from a worldsheet vantage point the effects of fluxes and gaugings.

Acknowledgments

A. The $k = 3$, $\mathcal{N} = 2$ minimal model

In order to work out the thresholds for the example of the quintic described by the $(k = 3)^5$ Gepner models it is helpful to decompose the primaries of the $k = 3$ minimal model ($c = 9/5$) into a $U(1)$ model ($c = 1$) combined with the 3 state Potts model, ($c = 4/5$). The primaries of the $U(1)$ model are $V_q = \exp(iq\sqrt{5/3}H)$ with $h = 5q^2/6$, where q is the charge. In the NS sector $q = n/5 = 2n/10$, while in the R sector $q = (2n + 1)/10$. The primaries of the 3 state Potts model (which is actually a quotient of the $c = 4/5$ minimal model wrt a spin 3 W symmetry) are six: I identity with $h = 0$, ϵ energy with $h = 2/5$ (real), σ and σ^* spins with $h = 1/15$, ρ and ρ^* parafermions with $h = 2/3 = (k - 1)/k$. Indeed the S-modular transformation reflects the Z_3 symmetry $S = S_3 \otimes S_2$ where S_3 is the S-matrix of $SU(3)$ at level 1 and

$$S_2 = \frac{2}{\sqrt{5}} \begin{pmatrix} s_1 & s_2 \\ s_2 & -s_1 \end{pmatrix} \tag{A.1}$$

sector	(l, m, s)	h	q	Field	Comment
NS	$(0, 0, 0)$	0	0	$V_0 I$	Identity
R	$(1, -2, -1)$	$3/40$	$+1/10$	$V_{+1/10}\sigma$	RGS
NS	$(2, 0, 0)$	$2/5$	0	$V_0 \epsilon$	
R	$(3, -2, -1)$	$27/40$	$+1/10$	$V_{+1/10}\rho$	
NS	$(1, +1, 0)$	$1/10$	$+1/5$	$V_{+1/5}\sigma^*$	CPO
R	$(0, -1, -1)$	$3/40$	$+3/10$	$V_{+3/10}I$	RGS
NS	$(3, +1, 0)$	$7/10$	$+1/5$	$V_{+1/5}\rho^*$	
R	$(2, -1, -1)$	$19/40$	$+3/10$	$V_{+3/10}\epsilon$	
NS	$(2, +2, 0)$	$1/5$	$+2/5$	$V_{+2/5}\sigma$	CPO
R	$(1, 0, -1)$	$11/40$	$+5/10$	$V_{+5/10}\sigma^*$	
NS	$(3, +3, 2)$	$4/5$	$+2/5$	$V_{+2/5}\rho$	
R	$(3, 0, -1)$	$7/8$	$+5/10$	$V_{+5/10}\rho^*$	
NS	$(3, +3, 0)$	$3/10$	$+3/5$	$V_{+3/5}I$	CPO
R	$(2, +1, -1)$	$19/40$	$+7/10$	$V_{+7/10}\sigma$	
NS	$(3, +1, 0)$	$7/10$	$+3/5$	$V_{+3/5}\epsilon$	
R	$(2, +1, -3)$	$43/40$	$+7/10$	$V_{+7/10}\rho$	
NS	$(1, +1, 2)$	$3/5$	$+4/5$	$V_{+4/5}\sigma^*$	
R	$(3, +2, -1)$	$27/40$	$+9/10$	$V_{+9/10}I$	
NS	$(3, +1, -2)$	$6/5$	$+4/5$	$V_{+4/5}\rho^*$	
R	$(1, -2, +3)$	$43/40$	$+9/10$	$V_{+9/10}\epsilon$	$+9/10 = $ -11/10(mod2)
NS	$(0, 0, 2)$	$3/2$	1	$V_{+1}R + V_{-1}\rho^*$	G (ws susy)
NS	$(2, 0, 2)$	$9/10$	1	$V_{+1}\sigma + V_{-1}\sigma^*$	

Table 1: The sectors of the $(k = 3)^5$ model.

where $s_n = \sin(n\pi/5)$. The resulting fusion rules also reflect this symmetry. In particular, I, ρ and ρ^* are simple currents. The only non obvious ones are

$$\rho \times \sigma = \sigma^* \quad , \quad \rho \times \epsilon = \sigma \quad , \quad \rho \times \sigma^* = \epsilon \tag{A.2}$$

and their conjugate, while ϵ, σ and σ^* have non abelian ('minimal' in a sense) fusion rules

$$\epsilon \times \epsilon = \sigma \times \sigma^* = I + \epsilon \tag{A.3}$$

as well as

$$\epsilon \times \sigma = \sigma^* \times \sigma^* = \rho + \sigma \tag{A.4}$$

and its conjugate.

In the Table we list the field identifications (barring charge conjugates).

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